

Single-Shot Generation and Detection of a Two-Photon Generalized Binomial State in a Cavity

R. Lo Franco,* G. Compagno, A. Messina, and A. Napoli
*Dipartimento di Scienze Fisiche ed Astronomiche,
 Università di Palermo, via Archirafi 36, 90123 Palermo, Italy*
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A “quasi-deterministic” scheme to generate a two-photon generalized binomial state in a single-mode high- Q cavity is proposed. We also suggest a single-shot scheme to measure the generated state based on a probe two-level atom that “reads” the cavity field. The possibility of implementing the schemes is discussed.

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Generation of nonclassical states of the electromagnetic field holds an important role in quantum optics both from the theoretical and experimental point of view. In fact, these states may give information about fundamentals of quantum theory and lead to applications in quantum information processing [1, 2]. Due to the experimental improvement of the quality factors of the cavities, Rydberg atoms lifetimes and control of the atom-cavity interactions [3, 4, 5, 6], cavity quantum electrodynamics (CQED) is particularly indicated for quantum field state engineering. In this context, several schemes have been proposed to generate, for example, Fock states using the interaction of consecutive atoms with a high- Q cavity [3, 7]. Recently a two-photon Fock state was generated and probed [8].

An important class of quantum non-classical states of the electromagnetic field is constituted by the binomial states, introduced by Stoler et al. [9], whose properties [9, 10, 11, 12] and interaction with atoms [13] have been studied. These states exhibit non-zero field expectation values, are characterized by a finite maximum number of photons and interpolate between the coherent state and the number state. They also have interesting applications. For example, binomial states have been proposed as reference field states in schemes to measure the canonical phase of quantum electromagnetic fields [14, 15]. Generation of entanglement between atoms and electromagnetic field was analyzed when a binomial state interacts with a mixed two-qubit system (two-level atoms) [16]. It was also recently shown that a binomial state gives an interesting transient spectrum when it constitutes the initial field state of a single-Cooper-pair box [17]. Thus, for its characteristic features and applications, it appears of interest to develop implementable procedures for the generation of binomial states.

A conditional scheme to generate binomial states in a cavity was proposed [18], in the CQED context, that exploits the quantum field state engineering in a single-mode cavity introduced by Vogel et al. [19]. This scheme utilizes the resonant interaction of N consecutive two-level atoms with the cavity initially prepared in its vacuum state. The desired cavity field state is then obtained

through a total state reduction by a measurement on the atoms coming out of the cavity. This scheme is conditional and results to have a low efficiency for generating binomial states with a maximum number of photons larger than one.

Here we propose an efficient, “quasi-deterministic” scheme for the generation and detection of a two-photon generalized binomial state in a single-mode high- Q cavity. Moreover, we discuss its implementation by considering the typical experimental errors involved in CQED systems.

Our generation scheme exploits two consecutive two-level atoms resonantly interacting, one by one, with the cavity initially prepared in its vacuum state. The interaction of each two-level atom with the single-mode cavity field is assumed to be well described by the Jaynes-Cummings Hamiltonian $H_{JC} = \hbar\omega\sigma_z/2 + \hbar\omega a^\dagger a + i\hbar g(\sigma_+ a - \sigma_- a^\dagger)$ [20], where ω is the cavity field mode, g the atom-field coupling constant, a and a^\dagger the field annihilation and creation operators and $\sigma_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$, $\sigma_+ = |\uparrow\rangle\langle\downarrow|$, $\sigma_- = |\downarrow\rangle\langle\uparrow|$ the pseudo-spin atomic operators, $|\uparrow\rangle$ and $|\downarrow\rangle$ being respectively the excited and ground state of the two-level atom. It is well known that the Hamiltonian H_{JC} generates the transitions [7, 21]

$$\begin{aligned} |\uparrow n\rangle &\rightarrow \cos(g\sqrt{n+1}t)|\uparrow n\rangle - \sin(g\sqrt{n+1}t)|\downarrow n+1\rangle \\ |\downarrow n\rangle &\rightarrow \cos(g\sqrt{n}t)|\downarrow n\rangle + \sin(g\sqrt{n}t)|\uparrow n-1\rangle, \end{aligned} \quad (1)$$

where $|\uparrow n\rangle \equiv |\uparrow\rangle|n\rangle$, $|\downarrow n\rangle \equiv |\downarrow\rangle|n\rangle$ and $a^\dagger a|n\rangle = n|n\rangle$.

It is useful, at this point, to recall that the single-mode generalized binomial state (GBS) with a maximum number of photons N , normalized and characterized by the probability of single photon occurrence $0 \leq p \leq 1$ and the mean phase ϕ is defined as [9, 10, 22, 23]

$$|N, p, \phi\rangle = \sum_{n=0}^N \left[\binom{N}{n} p^n (1-p)^{N-n} \right]^{1/2} e^{in\phi} |n\rangle. \quad (2)$$

This GBS is reduced to the vacuum state $|0\rangle$ when $p = 0$ and to the number state $|N\rangle$ when $p = 1$. In the particular case where $\phi = 0$, the GBS of Eq. (2) is named “binomial state” [9]. Here we concentrate on the 2GBS

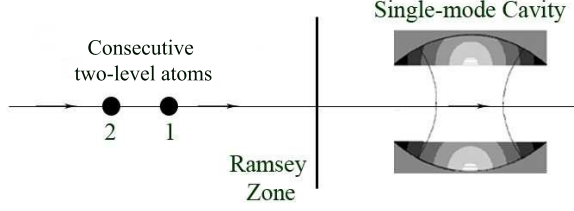


FIG. 1: Experimental setup for the generation of a two-photon generalized binomial state (2GBS) in a cavity.

$|2, p, \phi\rangle$ that can be explicitly written putting $N = 2$ in Eq. (2)

$$|2, p, \phi\rangle = (1-p)|0\rangle + \sqrt{2p(1-p)}e^{i\phi}|1\rangle + pe^{i2\phi}|2\rangle. \quad (3)$$

A sketch of our generation procedure is represented in Fig.1. Before entering the cavity, the k -th two-level atom ($k = 1, 2$) is injected into an appropriate Ramsey zone where it is prepared in the superposition

$$|\chi_k\rangle = \sqrt{p}|\uparrow\rangle + e^{i\varphi_k}\sqrt{1-p}|\downarrow\rangle \quad (k = 1, 2) \quad (4)$$

where $0 \leq p \leq 1$. The values of p and φ_k can be arbitrarily fixed by adjusting the Ramsey zone settings, i.e. the classical field amplitude and the atom-field interaction time. After crossing the Ramsey zone, the first atom resonantly interacts for a time T_1 with the cavity initially in the vacuum state $|0\rangle$. Using Eq. (1) together with Eq. (4) and choosing an interaction time $T_1 = \pi/2g$, we obtain the factorized total atom-cavity state $|\Psi_1(T_1)\rangle = e^{i\varphi_1}|p, \pi - \varphi_1\rangle|\downarrow\rangle$ where the cavity field state

$$|p, \pi - \varphi_1\rangle = \sqrt{1-p}|0\rangle + e^{i(\pi - \varphi_1)}\sqrt{p}|1\rangle, \quad (5)$$

is the GBS with $N = 1$ and mean phase $\phi = \pi - \varphi_1$, also called “generalized Bernoulli state” [9]. So, the field state inside the cavity is generated, in principle, exactly in a deterministic way. At this stage our scheme is identical to that one proposed in Refs. [18, 19]. It is worth to underline here that this ($N = 1$) GBS is measurable by a probe atom and this provides also a method to test a Bell’s inequality violation for an entanglement of these states in two separate cavities [2].

After the exit from the cavity of the first atom, i.e. after preparing the cavity in the state $|p, \pi - \varphi_1\rangle$, a time interval Δt will elapse before the second atom enters the cavity. During this time, the free field evolution induces a shift of the mean phase of the cavity field state equal to $-\omega\Delta t$. So, the second two-level atom finds the cavity field in the state $|p, \pi - \varphi'\rangle$ with $\varphi' = \varphi_1 + \omega\Delta t$. This second atom, prepared by the Ramsey zone in the superposition $|\chi_2\rangle$ of Eq. (4) with $\varphi_2 = \varphi'$, interacts with the cavity for a time T_2 . Exploiting once again Eq. (1) together with Eqs. (4), (5) and with φ' in place of φ_1 ,

at the end of the atom-cavity interaction the state of the total system can be written, within a global phase factor, as follows

$$\begin{aligned} |\Psi_2(T_2)\rangle = & \left[(\cos gT_2 - \sin gT_2)\sqrt{p(1-p)}e^{i(\pi - \varphi')}|0\rangle \right. \\ & + \cos(g\sqrt{2}T_2)pe^{2i(\pi - \varphi')}|1\rangle \Big] |\uparrow\rangle - \left[(1-p)|0\rangle \right. \\ & + (\sin gT_2 + \cos gT_2)\sqrt{p(1-p)}e^{i(\pi - \varphi')}|1\rangle \\ & \left. + \sin(g\sqrt{2}T_2)pe^{2i(\pi - \varphi')}|2\rangle \right] |\downarrow\rangle. \end{aligned} \quad (6)$$

It is immediate to observe that, if the following conditions were simultaneously satisfied

$$\sin(gT_2 + \pi/4) = 1, \quad \sin(g\sqrt{2}T_2) = 1, \quad (7)$$

then the state $|\Psi_2(T_2)\rangle$ would be reduced to a factorized state given by the product of the ground atomic state $|\downarrow\rangle$ and the 2GBS $|2, p, \pi - \varphi'\rangle$ of Eq. (3). Unfortunately the conditions of Eq. (7) cannot be simultaneously satisfied. However, we shall see that, by satisfying the first condition of Eq. (7), the function $\sin(g\sqrt{2}T_2)$ takes a value different from one for an amount δ smaller than the uncertainty due to the typical experimental errors. In fact, the first condition of Eq. (7) is satisfied for $gT_2 = \pi/4 + 2m_2\pi$, where m_2 is a non-negative integer. We now look for suitable values of m_2 such that the second condition of Eq. (7) is as near as possible to one. The typical experimental conditions limit the interaction times T in CQED systems inside the range $10^{-1} \leq gT \leq 10^2$ [3], and these in turns confine the possible values of m_2 inside the interval $0 \leq m_2 \leq 16$. Among these possible values of m_2 , we find numerically that the best approximation of the second condition of Eq. (7) occurs for $m_2 = 5$ which corresponds to $T_2 = 41\pi/4g$. For this value of T_2 we have $\sin(g\sqrt{2}T_2) = 1 - \delta$ where $\delta \sim 10^{-4}$. On the other hand, the deviation δ_{exp} , induced by typical experimental errors, may be estimated as $\delta_{exp} \approx 2(gT_2)^2(\Delta T_2/T_2)^2$ and, being $\Delta T_2/T_2 \sim 10^{-2}$ [4, 24], we have $\delta_{exp} \approx 10^{-1}$ hence $\delta \ll \delta_{exp}$. Summing up, in correspondence to the interaction time $T_2 = 41\pi/4g$, the cavity field state factor of $|\downarrow\rangle$ in Eq. (6) is given by

$$|\psi_{2,p,\pi - \varphi'}\rangle \equiv \frac{1}{\mathcal{N}_2} \sum_{n=0}^2 c_n^{(2)} [p^n(1-p)^{2-n}]^{1/2} e^{in(\pi - \varphi')}|n\rangle \quad (8)$$

where the coefficients are $c_0^{(2)} = 1$, $c_1^{(2)} = \sqrt{2}$, $c_2^{(2)} = 1 - \delta$ and $\mathcal{N}_2 \approx (1 - 2\delta p^2)^{1/2}$ is a normalization constant. In order to estimate how much the state $|\psi_{2,p,\pi - \varphi'}\rangle$ is near to the “target” 2GBS $|2, p, \pi - \varphi'\rangle$, we use the fidelity $\mathcal{F}(p) = |\langle 2, p, \pi - \varphi' | \psi_{2,p,\pi - \varphi'} \rangle|^2$. This tends to one when $p \rightarrow 0$, i.e. when the 2GBS is reduced to the vacuum state $|0\rangle$. However, for $p \neq 0$ the fidelity is near to one and for $p = 1/2$ we have $\mathcal{F}(1/2) \approx 1 - 1.6 \times 10^{-9}$. So, $|\psi_{2,p,\pi - \varphi'}\rangle$ can be effectively identified with the 2GBS $|2, p, \pi - \varphi'\rangle$ of Eq. (3). The probability \mathcal{P}_2 to generate the cavity field

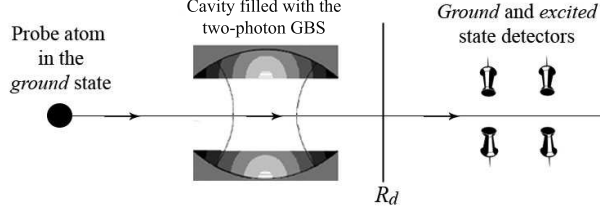


FIG. 2: Experimental setup for detecting the 2GBS inside the cavity. R_d is the “decoding” Ramsey zone.

state of Eq. (8) is equal to the probability of finding the second atom in the ground state $|\downarrow\rangle$ after coming out of the cavity and it results to be, at the time $T_2 = 41\pi/4g$, $\mathcal{P}_2 = |\langle\downarrow|\langle\psi_{2,p,\pi-\varphi}|\Psi_2(T_2)\rangle|^2 = \mathcal{N}_2^2$. Substituting the numerical value of \mathcal{N}_2 given above, the probability to generate the 2GBS is

$$\mathcal{P}_2 \approx 1 - 2 \times 10^{-4} p^2, \quad (9)$$

that is much higher than the analogous generation probability of previous conditional schemes ($\sim 1/4$) [18, 19]. Moreover, because of the high generation probability of the 2GBS of Eq. (9) and being the typical atomic detection efficiency less than one ($\sim 70\% \div 80\%$), the use of atomic detectors to collapse the total state to the target cavity state cannot further reduce the uncertainty on the generated cavity state itself. So, within the experimental limits, in our generation scheme a final atomic measurement is not required and it can be considered non-conditional: for this reason we name it “quasi-deterministic”.

At this point, in order to probe that the cavity is effectively filled with the 2GBS $|2, p, \phi\rangle$ of Eq. (3), we describe the following single-shot measurement scheme, illustrated in Fig.2. Let us consider the cavity prepared in the 2GBS $|2, p, \phi\rangle$ and a probe two-level atom prepared in the state $|\downarrow\rangle$ that resonantly interacts with the cavity for a time $T_P = 41\pi/4g$. We have seen that, for such a time, both the equalities of Eq. (7) can be retained satisfied within the experimental errors. Thus, using the Jaynes-Cummings evolutions reported in Eq. (1) together with Eq. (3), we find that, after the time T_P , the total state of the atom-cavity system is transformed as

$$|\downarrow\rangle|2, p, \phi\rangle \xrightarrow{T_P} |p, \phi\rangle(\sqrt{1-p}|\downarrow\rangle + \sqrt{p}e^{i\phi}|\uparrow\rangle), \quad (10)$$

where $|p, \phi\rangle$ is the GBS defined by Eq. (2) for $N = 1$. After coming out of the cavity the atom crosses a “decoding” Ramsey zone R_d set in such a way that it undergoes the following transformations

$$\begin{aligned} |\uparrow\rangle &\xrightarrow{R_d} \sqrt{p}|\uparrow\rangle - e^{-i\phi}\sqrt{1-p}|\downarrow\rangle \\ |\downarrow\rangle &\xrightarrow{R_d} e^{i\phi}\sqrt{1-p}|\uparrow\rangle + \sqrt{p}|\downarrow\rangle, \end{aligned} \quad (11)$$

with the values of p and ϕ coinciding with that ones defining the 2GBS $|2, p, \phi\rangle$ to be measured. Thus, utilizing

Eqs. (10) and (11), we find that after the Ramsey zone R_d the atom-cavity system undergoes the evolution

$$|\downarrow\rangle|2, p, \phi\rangle \xrightarrow{T_P, R_d} e^{i\phi}|p, \phi\rangle|\uparrow\rangle. \quad (12)$$

In this way, the measurement of the excited atomic state $|\uparrow\rangle$ at the end of the sequence of Fig.2 corresponds to the detection of the 2GBS $|2, p, \phi\rangle$ inside the cavity.

We stress that, considering the orthogonality property of binomial states [2], the 2GBS $|2, 1-p, \pi+\phi\rangle$, orthogonal to the previously generated 2GBS $|2, p, \phi\rangle$, can also be obtained by our generation scheme above with the changes $p \rightarrow 1-p$, $\varphi_k \rightarrow \pi + \varphi_k$ ($k = 1, 2$) in Eq. (4), which are achievable by appropriate adjustments of the Ramsey zone settings. On the other hand, if a probe two-level atom initially prepared in the ground state $|\downarrow\rangle$ finds the 2GBS $|2, 1-p, \pi+\phi\rangle$ inside the cavity and follows the same measurement scheme as above, the atom-cavity evolution is

$$|\downarrow\rangle|2, 1-p, \pi+\phi\rangle \xrightarrow{T_P, R_d} |1-p, \pi+\phi\rangle|\downarrow\rangle \quad (13)$$

and the measurement of the ground atomic state $|\downarrow\rangle$ at the end of the sequence of Fig.2 corresponds now to the detection of the 2GBS $|2, 1-p, \pi+\phi\rangle$ inside the cavity. We also note that the results of Eqs. (12) and (13) permit to distinguish, in a single-shot measurement, each of the two orthogonal 2GBSs $|2, p, \phi\rangle$ and $|2, 1-p, \pi+\phi\rangle$ inside the cavity.

The orthogonal 2GBSs $|2, p, \phi\rangle$, $|2, 1-p, \pi+\phi\rangle$ represent two vectors of an orthonormal basis in a 3-dimensional Fock-Hilbert space. The third vector of the basis can be readily obtained by setting the orthogonality and normalization conditions and it results to be

$$\begin{aligned} |\Gamma(2, p, \phi)\rangle &= \sqrt{2p(1-p)}|0\rangle + (2p-1)e^{i\phi}|1\rangle \\ &\quad - \sqrt{2p(1-p)}e^{i2\phi}|2\rangle. \end{aligned} \quad (14)$$

This state can be generated in a conditional way [25] by using the resonant interaction of an opportunely prepared two-level atom with a cavity filled with a GBS with $N = 1$. Moreover, utilizing the Holstein-Primakoff operators [26] for the case $N = 2$, $J_2^+ = \sqrt{2-\hat{n}}a$, $J_2^- = a^\dagger\sqrt{2-\hat{n}}$ and $J_2^0 = 1-\hat{n}$ with $\hat{n} = a^\dagger a$, we shall show that these three basis states, $|2, p, \phi\rangle$, $|\Gamma(2, p, \phi)\rangle$ and $|2, 1-p, \pi+\phi\rangle$ are eigenvectors of the pseudo angular momentum operator J_3 defined as

$$J_3 = \sqrt{p(1-p)}(e^{-i\phi}J_2^+ + e^{i\phi}J_2^-) - (2p-1)J_2^0. \quad (15)$$

In fact, using Eq. (15) and the explicit forms of the basis vectors given in Eqs. (3), (14), it is straightforward to prove the following eigenvalues equations

$$\begin{aligned} J_3|2, p, \phi\rangle &= |2, p, \phi\rangle; \quad J_3|\Gamma(2, p, \phi)\rangle = 0 \\ J_3|2, 1-p, \pi+\phi\rangle &= -|2, 1-p, \pi+\phi\rangle. \end{aligned} \quad (16)$$

So, using the usual notation $|l, m\rangle$ for the eigenvectors of angular momentum l , we can do the identifications

$|2, p, \phi\rangle \equiv |1, 1\rangle$, $|\Gamma(2, p, \phi)\rangle \equiv |1, 0\rangle$, $|2, 1 - p, \pi + \phi\rangle \equiv |1, -1\rangle$, and we can describe the 3-dimensional Fock-Hilbert subspace as a subspace of angular momentum $l = 1$ spanned by the basis $\mathcal{B} = \{|1, 1\rangle, |1, 0\rangle, |1, -1\rangle\}$. In this context, we say that our measurement scheme of orthogonal 2GBSs, illustrated in Fig.2 with the results given by Eqs. (12), (13), constitutes a method to measure the eigenvalues ± 1 of the field operator J_3 of Eq. (15).

We now discuss the possible implementation of our quasi-deterministic schemes. In our generation scheme the atom-cavity interaction times are different for each atom, respectively T_1, T_2 . These can be obtained either by selecting opportune different velocities for each atom or by selecting the same velocity for the two atoms and applying an electric field inside the cavity in order to Stark shift each atom out of resonance so to obtain the desired resonant interaction time [27]. The appropriate atomic velocity may be selected by laser induced atomic pumping [4]. The experimental uncertainties of the selected velocity Δv and interaction time ΔT are such that $\Delta T/T \approx \Delta v/v$. In current laboratory experiments it is possible to select a given atomic velocity such that $\Delta v/v \sim 10^{-2}$ or less [4, 24]. In our generation and measurement schemes, we have also ignored the atomic or photon decay during the atom-cavity interactions. This assumption is valid if $\tau_{at}, \tau_{cav} > T$, where τ_{at}, τ_{cav} are the atomic and photon mean lifetimes respectively and T is the interaction time. For Rydberg atomic levels and microwave superconducting cavities with quality factor $Q \sim 10^8 \div 10^{10}$ the required condition on the mean lifetimes can be satisfied, because $\tau_{at} \sim 10^{-5} \div 10^{-2}$ s, $\tau_{cav} \sim 10^{-4} \div 10^{-1}$ s and $T \sim 10^{-5} \div 10^{-4}$ s [3]. Moreover, the typical mean lifetimes of the Rydberg atomic levels τ_{at} must be such that the atoms do not decay during the entire sequence of the schemes [3, 4].

In conclusion, we have shown that it is possible to generate, within the experimental errors in a non-conditional way, a two-photon generalized binomial state (2GBS) inside a single-mode cavity by using two consecutive two-level atoms interacting with the cavity each for a given time. Moreover, the presence inside the cavity of the 2GBS can be verified by a single-shot measurement scheme utilizing a probe two-level atom, prepared in its ground state, that resonantly interacts with the cavity for a given time and “reads” the cavity field state. The information acquired by the probe atom is then “decoded” by a suitable Ramsey zone and finally “read” by measuring the internal atomic state. The results of this work therefore open the way to generation and detection schemes of superpositions of two orthogonal 2GBSs in a cavity (“binomial Schrödinger cat”) or entangled 2GBSs in separate cavities [25], which can be useful both for investigations of the foundations of quantum theory and for applications in quantum information processing. An extension of the scheme proposed here is possible for an efficient

generation of GBSs with $N > 2$ and it will be treated somewhere else. At this time, the experimental developments seem to be rather promising on the possibility of implementing our schemes.

* Electronic address: lofranco@fisica.unipa.it;
URL: <http://www.fisica.unipa.it/~lofranco>

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